Comparison of Energy Performance of Commonly Used Insulation Configurations for Slab-on-Grade Floors

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INTRODUCTION AND BACKGROUND

To reduce ground-coupling heat losses from single-family houses, three of the approaches used to insulate slab-on-grade floors are vertical insulation placed along the foundation wall, horizontal insulation placed under the slab perimeter, and horizontal insulation extended on the building exterior. The first two approaches are common in the design of insulation systems for slab-on-grade foundations throughout the United States.

In this paper, the thermal performance of each of the above-mentioned insulation configurations is evaluated when a given amount of insulation is utilized. A thermally optimal insulation configuration is then determined for a given slab-on-grade floor, location, insulation amount (or length), and insulation U-value (or thickness). It is important to note that the problem of finding the optimal insulation configurations —which is one of the goals of this paper—is different from the problem of obtaining the optimal insulation distribution. The problem of determining the insulation distribution with the slab-on-grade floor that minimizes total heat loss is discussed in detail by Claesson and Eftring (1980). They showed that to obtain "optimal" distribution, the thickness of the insulation has to vary continuously along the slab surface. Because of the continuous variation of its insulation thickness, the optimal distribution configuration is too expensive to install for slab-on-grade floors. However, Labs (1985) proposed an approximate optimal distribution that is probably cheaper to install. This approximate distribution uses a stepped configuration of several layers of insulation boards of uniform thickness.

The ground-coupled heat transfer calculation method used for this study is the interzone temperature profile estimation (ITPE) technique developed by Krarti and others (Krarti 1989; Krarti et al. 1988a, 1988b, 1990). The ITPE method combines analytical and numerical techniques to arrive at the functional form of the solution of the heat conduction equation in the ground surrounding the slab-on-grade floor. Soil temperature field distribution, heat flux variation along the slab surface, and total slab heat losses are obtained to analyze the heat transfer between the slab and the ground.

The first section of this paper describes the recommended construction practices in slab-on-grade foundation design related to the insulation location. The advantages and/or disadvantages of each insulation configuration are summarized. The second section presents the outlines of the energy calculation method needed to generate the data used in this study. The temperature field distribution around the slab-on-grade floor for each insulation configuration, the heat flux variation, and total slab heat losses are presented in the third section.

Common Insulation Configurations

Figure 1 shows the three insulation configurations commonly encountered in slab-on-grade construction. In the first (Figure 1a), the insulation is placed vertically along the foundation wall on either the interior or the exterior side. In both approaches, the insulation length is limited by the footing wall depth. The advantage of exterior insulation is that it protects the footing wall from the surrounding soil. A protective coating of flashing is needed, however, for the exposed area of exterior insulation to avoid physical damage and the sun's deleterious effects. This coating is not needed if the insulation is placed on the interior side of the foundation wall. However, interior insulation is more difficult to install since insulation needs to be placed in the joint between the slab and the foundation wall to avoid thermal bridging (Labs et al. 1988).

The thermal insulation can be placed outside the building, extending horizontally into the surrounding soil (Figure 1b). In this configuration, the insulation is vulnerable to frost and moisture. Therefore, only insulation materials such as extruded or expanded polystyrene should be used. Indeed, polystyrene does not degrade significantly when exposed to moisture or frost.

The most common insulation configuration places insulation along the slab either on top of or under the slab, as shown in Figure 1c. The insulation is generally placed on the perimeter of the slab, although the entire slab may be insulated in very cold climates. In this insulation configuration, the foundation wall is exposed to the inclement soil climate. When it is under the slab, the insulation must

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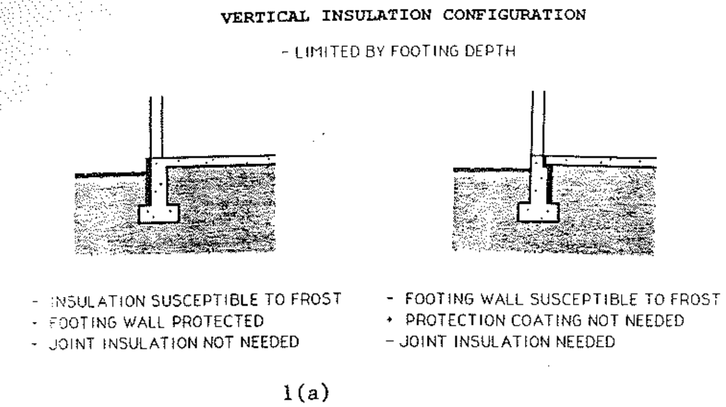

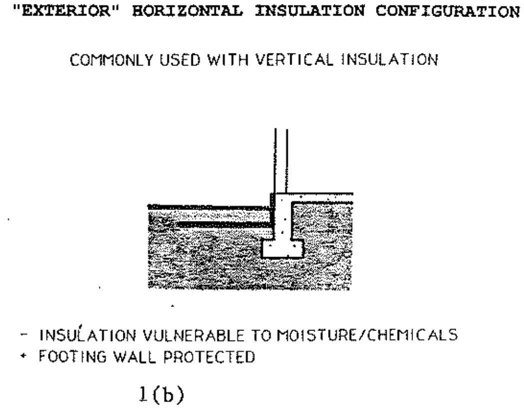

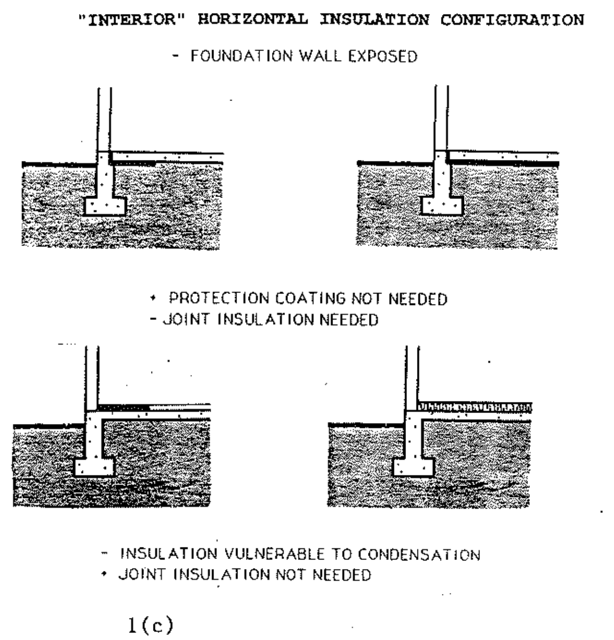

Figure 1 *Common insulation configurations for slab-on-grade floors.*

be placed at the joint slab-foundation wall. This is not needed when the insulation is placed above the slab, but, in this configuration, the frost depth may be greater in the vicinity of the slab edge.

As mentioned in the introduction, the insulation can be placed nonuniformly under or above the slab to minimize energy losses to the ground. This is the case in the optimal insulation distribution as derived and analyzed by Claesson and Eftring (1980). In this paper, the optimal insulation distribution configuration will be included only for comparison purposes. The conventional insulation configurations, which will be analyzed in the following section, are (a) uniform insulation under the entire slab, (b) perimeter insulation extending 1 m under the slab, (c) exterior insulation extending 1 m outside the slab, and (d) vertical insulation with a 1-m depth.

The thermal performance of the above-described insulation configuration, including the optimal insulation distribution, will be calculated using the two-dimensional interzone temperature profile estimation (ITPE) technique. The general outlines of the ITPE technique will be described in the following section. For more details on the application of ITPE to slab-on-grade floors refer to Krarti (1989) and Krarti et al. (1988a, 1988b, 1990).

General Calculation Procedure

Figure 2 presents a typical model for heat transfer analysis of slab-on-grade floors. Note that the figure does not show slab foundation details, such as foundation walls and footings, which are generally made up of poured or precast concrete. Typically, the thermal conductivity of the ground is similar to that of precast concrete. Therefore, thermally the foundation elements of the floor can be considered as integral parts of the ground medium. The model of Figure 2 can then be applied when insulation is placed on the interior (i.e., on top) of the slab-on-grade floor. Using more elaborate models, other ground-coupled heat loss calculation methods predicted virtually no difference in energy performance between exterior and interior insulation placements (Labs et al. 1988; Mitalas 1987).

The time-dependent Laplace equation governing heat transfer between slabs and the ground can be reduced to a time-independent Helmholtz equation using the complex temperature amplitude technique (Carslaw and Jaeger 1959). The general form of the Helmholtz equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \delta^2 T \tag{1}$$

SOLUTION PROCEDURE

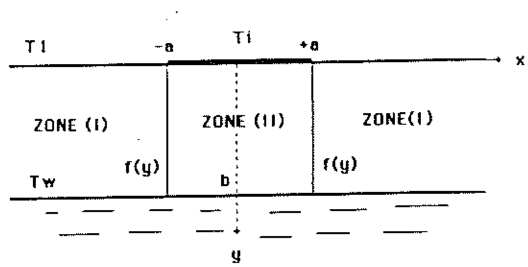

Figure 2 *Slab-on-grade floor model.*

where T is the complex temperature amplitude as a function of the space coordinates x and y, and δ is a complex variable function of temperature angular frequency and soil thermal diffusivity (Krarti et al. 1988b). The boundary conditions of Equation 1 depend on the insulation configuration of the slab-on-grade floor. The complete governing equations are given in Krarti (1989) and Krarti et al. 1988a, 1988b, 1990). In all cases, a third boundary condition is used at the slab-soil interface. This boundary condition expresses the continuity of heat flux between the lower slab surface and the interior air via (1) the convective film coefficient at the inside surface, (2) the insulation conductance, (3) the slab material conductance, and (4) the interface contact conductance between slab and soil.

The ITPE technique, combined with the Fourier series, is used to solve the Helmholtz equation. In the ITPE technique, the ground is divided into several zones of rectangular shape. In the case of the slab-on-grade floor configuration shown in Figure 2, the ground is divided into three zones by the surfaces $x = \pm$ a, the central zone (zone II), and the two symmetrical zones (zone I). In each zone, the temperature is analytically determined by solving the Helmholtz equation. Note that along at least one side of each zone, the temperature profile f (e.g., $T[\pm a,y] = f(y)$ in Figure 2) is not known. From Fourier series theory, the profile f can be determined if its Fourier coefficients, f_n, are known. The condition of temperature gradient continuity between zones gives a linear system of equations for which the Fourier coefficients, f_n, are the unknown. When all the profiles of type f are taken into consideration, the resulting linear systems of equations can be condensed into the following form:

$$[A]\{F\}=B \tag{2}$$

where A is a square $N \times N$ matrix of the general term $A_{i,j}$. This matrix depends solely on geometric dimensions, insulation values, and frequency. It characterizes the slab configuration. $\{F\}$ is a column matrix of the coefficients f_n, $n = 1, \ldots N$. It contains the Fourier coefficients for the profile f. $\{B\}$ is a column matrix of the general term B_j and is a linear function of the indoor temperature, T_i, the water table, T_w, and the soil surface temperature, T_s:

$$\{B\} = \{B_i\}T_i+\{B_w\}T_w+\{B_s\}T_s \tag{3}$$

The expression of the general term of the matrices $[A]$ and $\{B\}$ specific to slab-on-grade floors is derived and given in Krarti (1989) and Krarti et al. (1988a, 1988b, 1990).

To determine the Fourier coefficients, f_n, the matrix $[A]$ needs to be inverted. There are a number of techniques for inverting square matrices and solving linear systems. In this paper, the system of equations is solved using the Jordan-Gauss elimination method.

Once the Fourier coefficients of all the profiles X are calculated, the boundary conditions for each zone become known and the temperature can be determined throughout the soil. The total heat flux, Q, to the ground from the building envelope can be readily obtained from the temperature expressions. The general form of Q is

$$Q = \sum_{n=1}^{N} q_n(f_n,T_i,T_s,T_w) \ . \tag{4}$$

The general term, q_n, of the above equation is proportional to $1/n^2$, which is a rapidly convergent series.

It is interesting to note that Q is a linear function of the temperatures T_i, T_w, and T_s:

$$Q = Q_iT_i + Q_wT_w + Q_sT_s \tag{5}$$

In the particular case where the temperatures T_i and T_w are constant throughout the year, the amplitude of the heat loss function, Q_s, is proportional to the soil surface temperature amplitude, T_s. For steady-state conditions, the heat flux Q can be expressed as

$$Q = Q_w(T_w-T_i) + Q_s(T_s-T_i) \ . \tag{6}$$

Moreover, as the water table level decreases, the coefficient Q_w approaches zero and the steady-state heat losses, Q, become directly proportional to the temperature difference across the building envelope (i.e., T_s - T_i).

As is evident from Equation 1, the model used throughout this paper assumes two-dimensional heat flow with homogenous soil properties. Therefore, moisture diffusion and corner effects are not accounted for. In addition, the model considers only the losses to the ground and therefore ignores thermal bridging effects such as those occurring at the slab edge-exterior wall joints. However, the ITPE formalism has been applied to three-dimensional slab-on-grade models (Krarti et al. 1990) and can be used to include thermal bridging effects.

SUMMARY OF RESULTS

Temperature Field

Figure 3 shows the annual average temperature distribution beneath a slab-on-grade floor with a half-width of 5 m (16.4 ft). The interior air of the house above the slab has a temperature of 20°C (68°F), while the temperature of the soil surface and water table is assumed to be 10°C (50°F). The water table level is 5 m (16.4 ft) deep. The soil has a

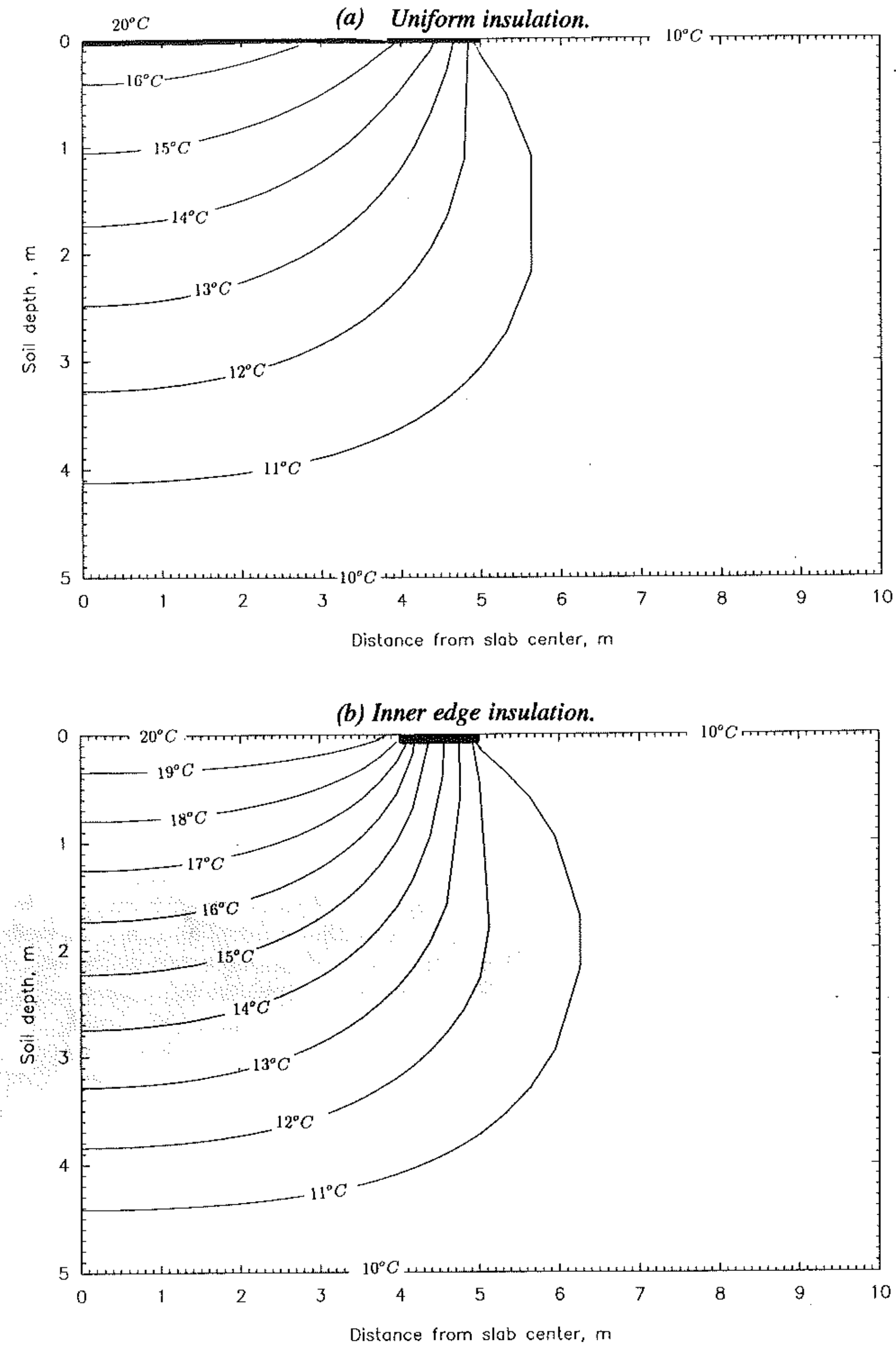

Figure 3 *Annual average earth temperature beneath a 10-m-wide slab with 5-m-deep water table. (a) Uniform insulation. (b) Inner edge insulation. (c) Outer edge insulation. (d) Vertical insulation. (e) Optimal insulation distribution.*

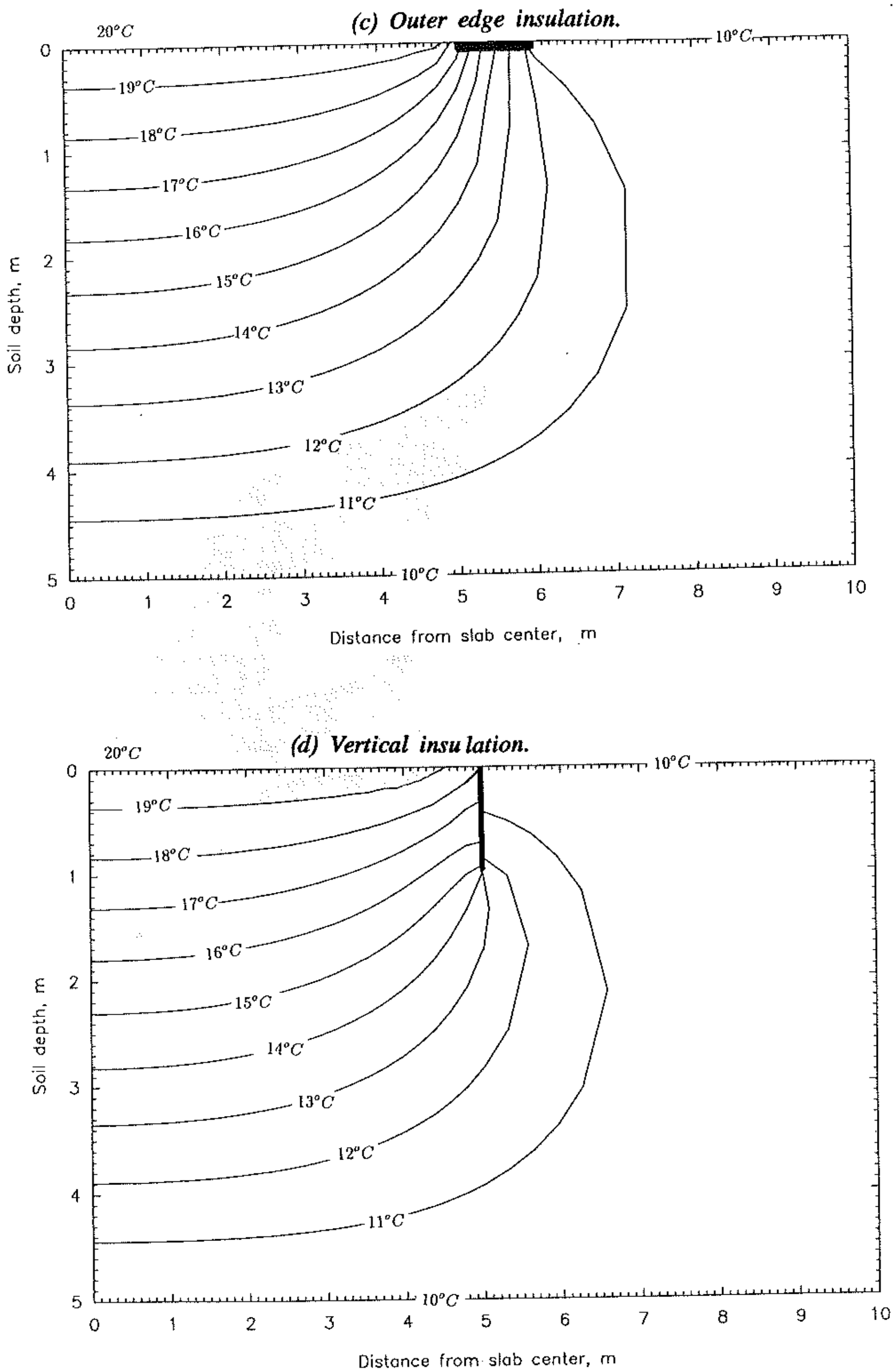

Figure 3 Continued

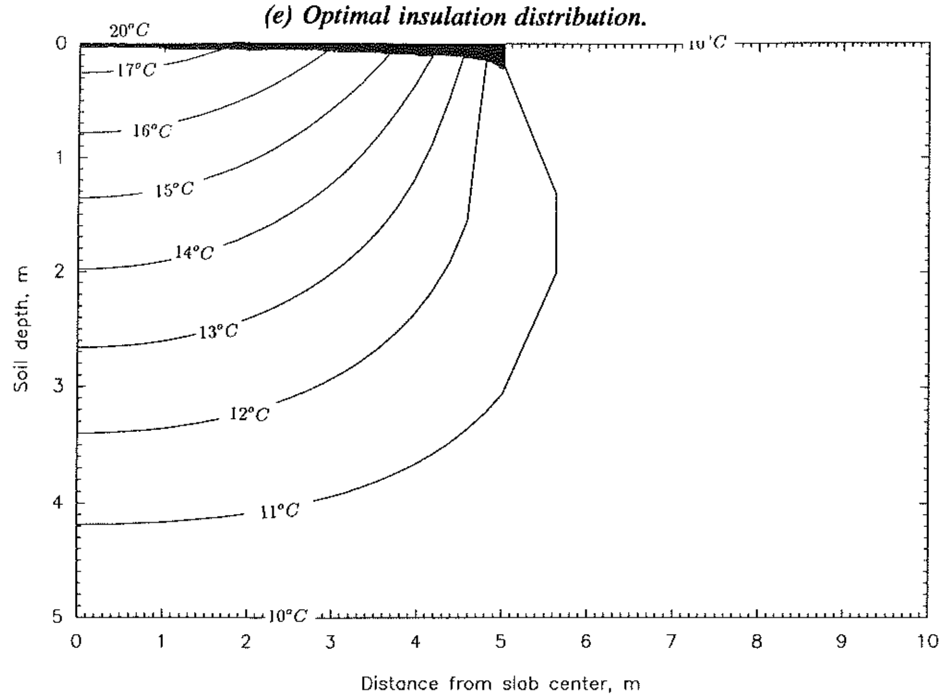

Figure 3 Continued

thermal conductivity of 1.0 W/m·°C (0.6 Btu/h·ft·°F) and a thermal diffusivity of 6.95 10^{-7} m^2/s (0.74 ft^2/h).

When R=2 m^2·°C/W (R = 11.4 ft^2·h·°F/Btu), insulation is uniformally placed along the slab floor, and the soil temperature varies as indicated in Figure 3a. In particular, the slab temperature decreases from about 16.5°C (61.7°F) at the center to 10°C (50°F) at the edge. At the edge of the slab, the isotherms become very close, resulting in a higher temperature gradient and therefore higher heat loss from the slab perimeter.

If the same amount of insulation is placed on the perimeter with the insulation extending inward 1 m (3.28 ft), the equivalent thermal resistance at the slab perimeter will be R = 10 m^2 ·°C/W (R = 57 ft^2 ·h·°F/Btu). Figure 3b shows the soil temperature variation when insulation of R = 10 m^2·°C/W (R=57 ft^2·h·°F/Btu) is placed at the perimeter of the slab extending inward 1 m. The central area of the slab is left uninsulated. In this configuration, the slab temperature decreases from almost 20°C (68°F) at the center to 10°C (50°F) at the edge.

When placed at the outer edge of the slab, the R=10 m^2·°C/W (R=57 ft^2·h·°F/Btu) thermal insulation changes the soil temperature field as indicated in Figure 3c. In this configuration, the entire slab is kept uninsulated, while a 1-m (3.28 ft) strip of the ground surface immediately adjacent to the slab perimeter is insulated. As shown in Figure 3c, the slab is relatively warm and its temperature varies from 20°C (68°F) at the center to 17.5°C (63.5°F) at the perimeter.

Figure 3d shows the soil temperature distribution when the R=10 m^2·°C/W (R=57 ft^2·°F/Btu) insulation is placed vertically along the foundation wall. The slab is kept uninsulated. As indicated in Figure 3d, the temperature isotherms "slide" down the vertical insulation, resulting in a warm under-slab surface and inner side of the foundation wall. Indeed, the slab temperature varies from 20°C (68°F) at the center to about 19°C (64.4°F) at the perimeter.

Finally, Figure 3e presents the temperature field beneath the same slab described above but insulated to minimize total slab heat losses. The soil temperature variation obtained with this "optimum" insulation distribution is similar to that found when the insulation is placed uniformly along the slab (see Figure 3a). The central area of the slab is a little warmer, with a temperature of about 17.5°C (63.5°F).

Heat Flux Distribution

Figure 4 shows the thermal insulation distribution along the horizontal surfaces of the slab and ground for the configurations described above. Note that the area beneath all the curves presented in Figure 4 is the same and is proportional to the thickness and length of the thermal insulation used. This area represents the amount of insulation placed above the slab or soil surface. Figure 5 summarizes the heat flux distribution along the slab surface for all the insulation configurations analyzed in the above section. When the optimal insulation distribution is used, the heat loss from the slab is uniform: the same amount of heat is lost from the perimeter as from the center of the slab. In fact, Claesson and Eftring (1980) used a special concept of thermodynamics—the "thermality"—to show that the

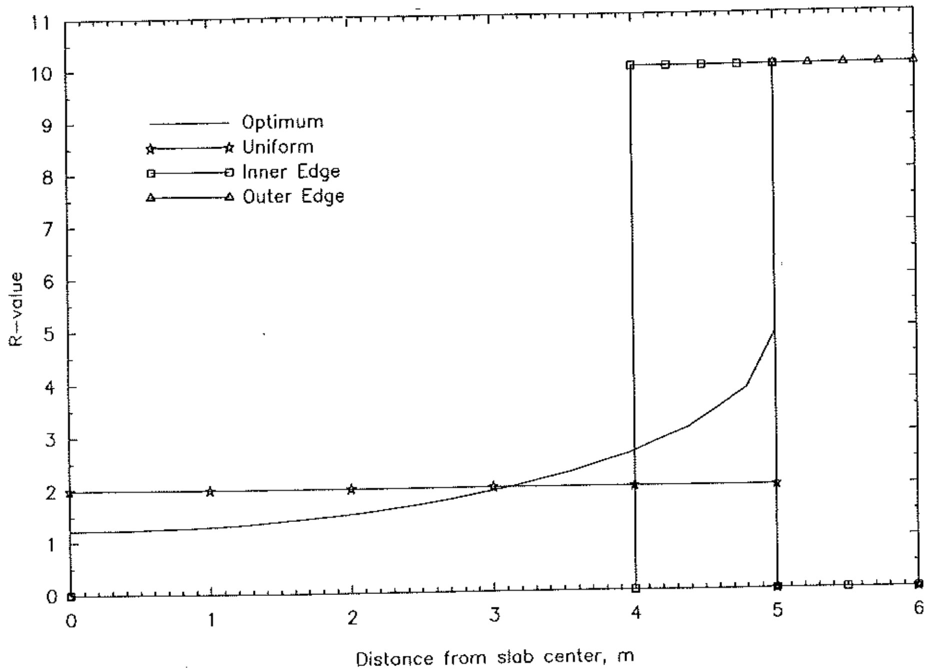

Figure 4 ***Insulation distribution along a 10-m-wide slab with an average R-value = 2.***

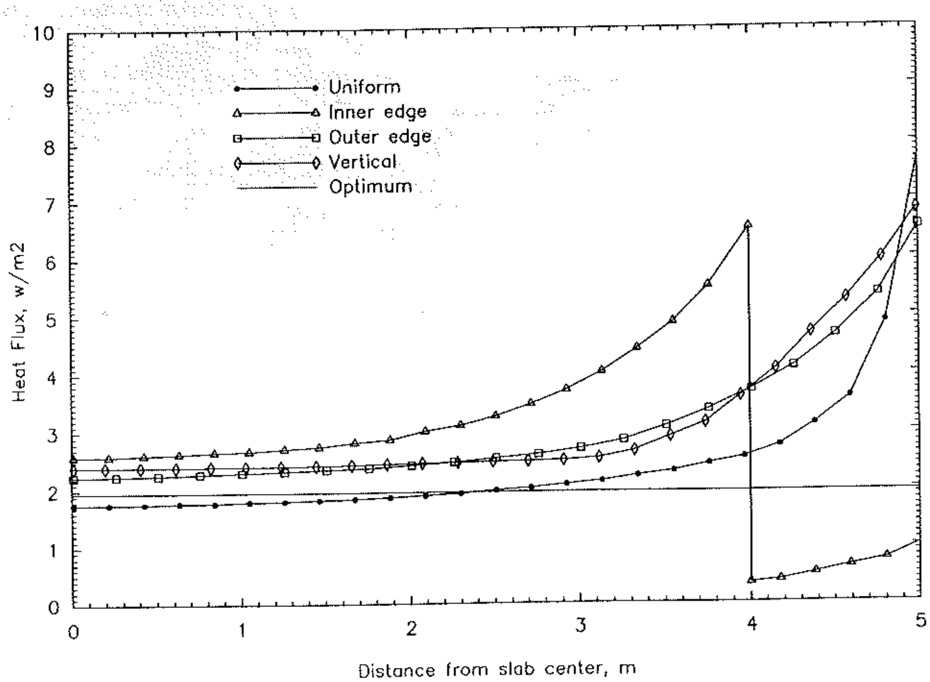

Figure 5 ***Heat flux distribution along a 10-m-wide slab with an average R-value = 2.***

criterion for optimal insulation distribution is a constant heat flux along the slab and, in general, along a ground-coupled surface. In almost all other configurations, the perimeter heat losses are higher than those at the center of the slab. When the insulation is placed solely at the perimeter, the heat losses are reduced significantly at the slab where the insulation is placed. However, in the area of the slab located between 3 m (9.8 ft) and 4 m (13.1 ft) from the slab center, the heat loss is significantly higher for the perimeter insulation configuration than for all the other configurations. For the central area of the slab, the heat loss is the lowest when the insulation is placed uniformly along the slab.

Figure 6 shows the R-value distribution along the horizontal surfaces of the slab and ground for all configurations (except the vertical insulation) when the amount of thermal insulation is small, with an average R-value of 0.2 $m^2 \cdot °C/W$ (R = 1.14 $ft^2 \cdot h \cdot °F/Btu$). Thus, if the insulation is placed uniformly along the slab, its R-value is 0.2 $m^2 \cdot °C/W$ (R = 1.14 $ft^2 \cdot h \cdot °F/Btu$). However, when the insulation is placed on a 1-m (3.28 ft) strip around the slab edges, its R-value becomes 1.0 $m^2 \cdot °C/W$ (R = 5.7 $ft^2 \cdot h \cdot °F/Btu$). Figure 6 shows that for the optimum insulation configuration, the insulation is concentrated on the perimeter of the slab. At the center of the slab, no insulation is needed to ensure optimum heat losses from the entire slab.

Figure 7 presents the heat flux variation along the slab when the insulation is placed according to the distributions indicated in Figure 6. The vertical insulation is placed along 1 m (3.28 ft) of the foundation wall with R-value = 1 $m^2 \cdot °C/W$ (R = 5.7 $ft^2 \cdot h \cdot °F/Btu$). All non-optimal insulation configurations have higher heat losses at the perimeter of the slab than the optimal insulation configuration. However, at the center of the slab, the heat loss is highest when the insulation is placed according to the optimum distribution.

Total Slab Heat Losses

Figure 8 shows the monthly total slab heat loss variation when the thermal resistance corresponds to an average R-value of 2 $m^2 \cdot °C/W$ (R = 11.4 $ft^2 \cdot h \cdot °F/Btu$). This amount of insulation is commonly used in cold climates. In Figure 8, the air temperature of the house above the slab is kept constant at 20°C (68°F). The soil surface temperature varies sinusoidally, with an average of 10°C (68°F) and amplitude of 5°C (9°F). The water table at 5 m (16.4 ft) below the slab has a constant temperature of 10°C (50°F). As shown in Figure 8, during most of the year, uniform insulation configuration gives the lowest total slab heat losses. However, during the winter, months the energy performance of the vertical insulation and outer perimeter insulation configurations is comparable to that of the uniform insulation configuration. For the vertical insulation and outer perimeter insulation configurations, the amplitude of the total heat loss is significantly lower than that of the uniform and inner edge insulation configurations.

Figure 9 shows the monthly total slab heat loss variation when the thermal resistance has an average R-value of 0.2 $m^2 \cdot °C/W$ (R = 1.14 $ft^2 \cdot h \cdot °F/Btu$). As mentioned earlier, if this insulation is placed on a 1 m (3.28 ft) strip around the slab perimeter, its R-value becomes 1.0 $m^2 \cdot °C/W$ (R = 5.7 $ft^2 \cdot h \cdot °F/Btu$). This amount of insulation is typically used in moderate climates. During winter months, the heat losses from the slab are lowest for the outer edge insulation configuration. However, during summer months,

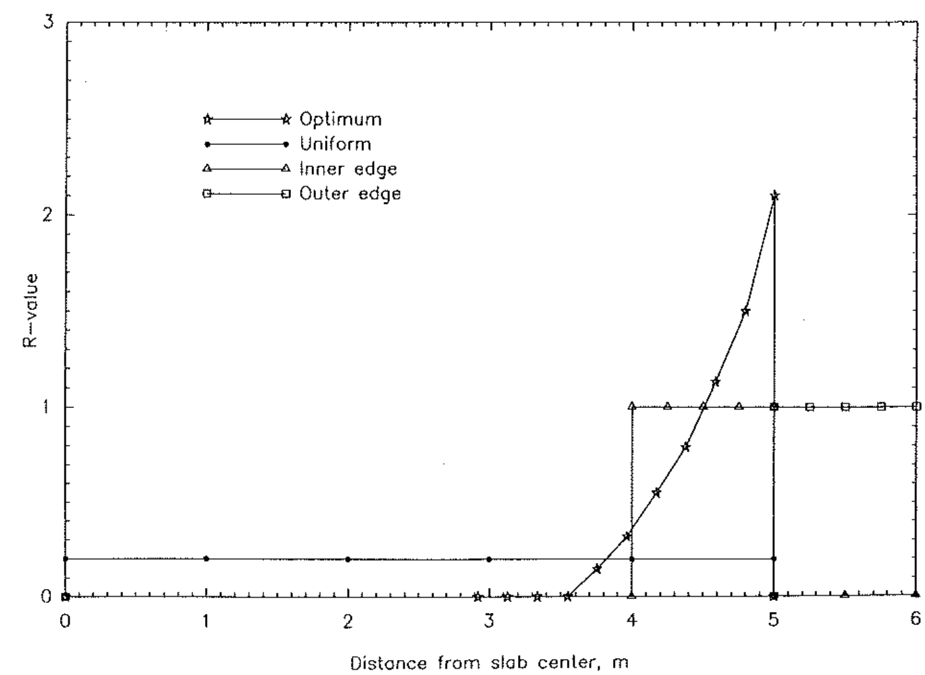

***Figure 6** Insulation distribution along a 10-m-wide slab with an average R-value = 0.2.*

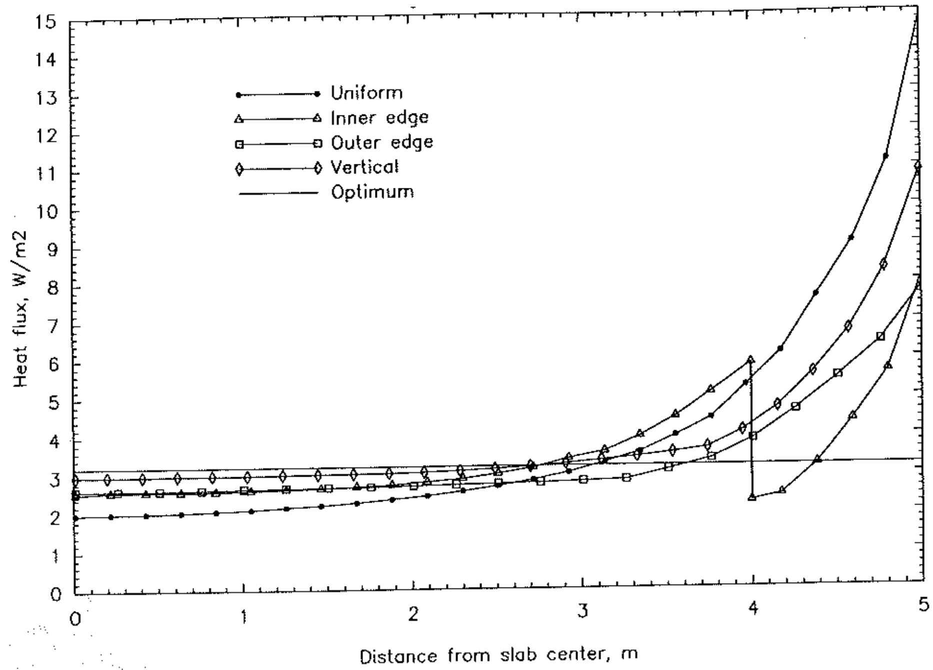

Figure 7 ***Heat flux distribution along a 10-m-wide slab with an average R-value = 0.2.***

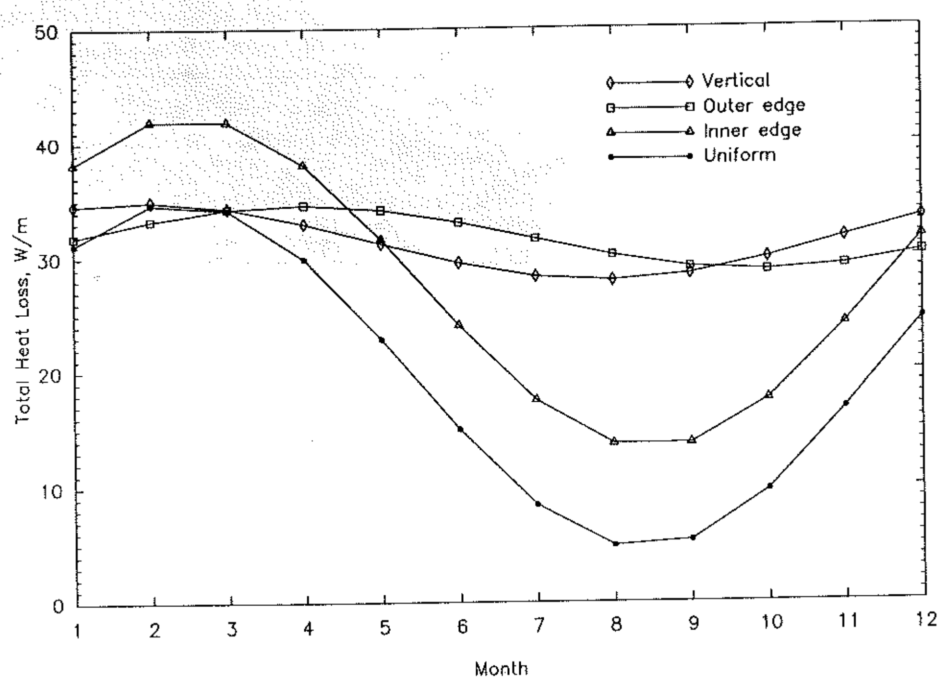

Figure 8 ***Monthly total heat loss from a 10-m-wide slab with an average RSI-value = 2.***

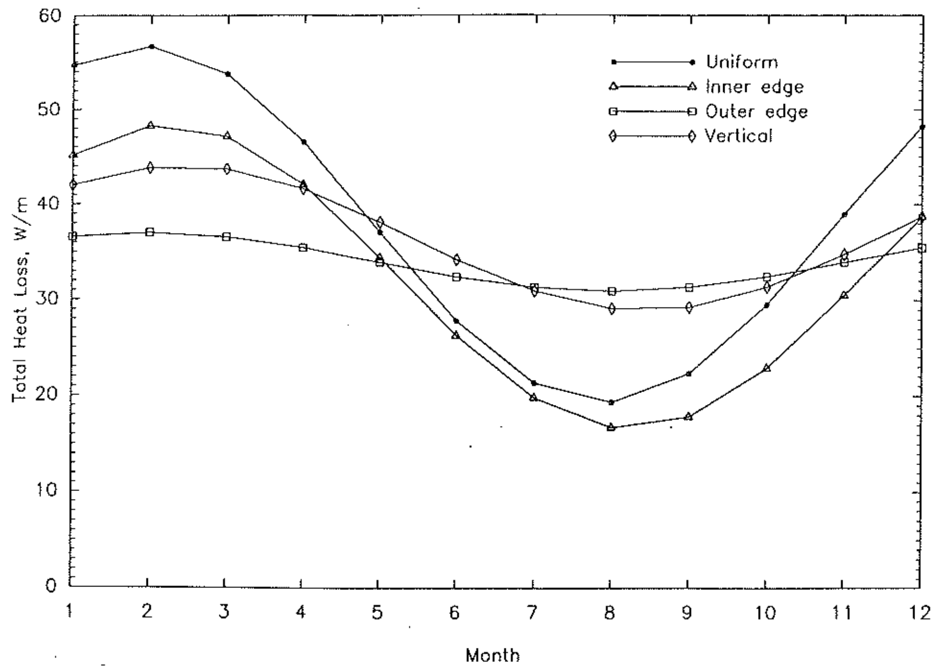

Figure 9 *Monthly total heat loss from a 10-m-wide slab with an average RSI-value = 0.2.*

the total slab heat losses are lowest for the inner edge insulation configuration. The uniform insulation configuration gives higher heat losses than the inner edge insulation configuration. This result is reversed when a higher amount of insulation is used, as is the case in Figure 8. A "shielding" effect of the vertical and outer edge insulation can be seen in Figures 8 and 9. Indeed, the total heat loss has a very low amplitude for both vertical and outer edge insulation configurations.

CONCLUSION

The thermal performance of commonly used insulation configurations for slab-on-grade floors has been analyzed. The result of this analysis indicates that for very cold climates, where a large amount of insulation is needed, it is more effective to place the insulation uniformly along the entire slab surface. However, for moderate climates, where the amount of insulation required is not significant, it is better to place the insulation at the inner perimeter of the slab. It is important to remember that this discussion is based solely on an approximate two-dimensional heat transfer analysis. Moreover, to determine the optimal insulation configuration for a given slab-on-grade floor, an economic analysis is required that takes into consideration a number of economic factors.

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